Magnetoplasma oscillations of the wave-guided type in semiconductor superlattices

1990 J. Phys.: Condens. Matter 2 6279

(http://iopscience.iop.org/0953-8984/2/29/005)

J. Phys.: Condens. Matter 2 (1990) 6279–6286. Printed in the UK

Magnetoplasma oscillations of the wave-guided type in semiconductor superlattices

J C Granada†, A M Kosevich‡ and Yu A Kosevich§
† International Centre for Theoretical Physics, 34100, Trieste, Italy
‡ Institute for Low Temperature Physics and Engineering, 310164 Kharkov, USSR
§ All-Union Surface and Vacuum Research Centre, 117334 Moscow, USSR

Received 20 November 1989, in final form 29 March 1990

Abstract. The dispersion relation for the electromagnetic wave-guided oscillations of a two-dimensional (2D-) electron layer periodic array in a transversal magnetic field is analysed. Oscillation frequencies much lower than the electron cyclotron frequency exist under certain conditions. Additionally, the perpendicular propagation to the layers is investigated. It is demonstrated, that a strong magnetic field causes a frequency shift and splitting, which are in inverse relation to the external magnetic field and the period of the layered electron system.

1. Introduction

The spectrum of electronic collective excitations in semiconductor superlattices has received much attention both from a theoretical as well as from an experimental point of view [1, 2]. In particular, much attention has been focused on elementary excitations in semiconductor supperlattices in an external magnetic field [3–10]. The dispersion relation of the cyclotron waves for a layered electron system in the Voigt configuration with a static magnetic field perpendicular to the planes was derived in [3]. Tselis and co-workers [4] obtained the dispersion relation for the frequency of collective excitations in a superlattice as a function of the components of the wave vector parallel and perpendicular to the layers. The microscopic quantised approach of magnetoplasmons given in [4] was compared with the semiclassical description of plasma excitations in a multilayer system with finite layer electron thickness [5]. As a result of [4, 5], the existence of helicon waves propagating along the superlattice axis was demonstrated. The presence of these low-frequency modes was previously suggested in [6] on the basis of experimental studies of magnetic field dependence of far infrared transmission through highly doped InAs/GaSb superlattices.

Helicons in multilayer systems have frequencies below the cyclotron frequency of the low-dimensional carriers and exhibit branch-folding effects due to the macroscopic periodicity of the superlattice. In the low frequency regime the helicon frequency is inversely proportional to the Hall conductivity of the 2D-electron gas, i.e., the helicon frequency is *proportional* to the applied magnetic field. Under the conditions of the quantum Hall effect, plateaus in the frequency of the helicon resonance should be observed [4]. In this case low-frequency helicon modes behave like helicons propagating

0953-8984/90/296279 + 08 $03.50

in a homogeneous medium, but showing undamping [7]. The role of collision-induced instabilities [8], of non-local effects [9] and of polarisation and displacement currents on the helicon wave propagation [10] have also been considered.

In experimental investigations of the high frequency absorption by a 2D electron gas trapped on the liquid ^{4}He surface, electromagnetic resonance frequencies much lower than the electron cyclotron frequency and depending *inversely* on the external magnetic field were observed [11]. A model of 'edge' magnetoplasmons was proposed for the theoretical explanation of these resonances [12]. It was shown, that 'edge' magnetoplasmons can propagate in systems containing inhomogeneous 2D electron gas.

On the other hand, in previous papers [13] the spectrum of magnetoplasma wave-guided oscillations of a *single* 2D homogeneous electron layer in a bounded system was described. It was demonstrated that a strong magnetic field causes a frequency shift and splitting, depending inversely on the external magnetic field. The fact that such a mode exists in an unbounded system along the layer plane electron system tells us that this magnetoplasma mode has a different physical origin from the edge magnetoplasmon.

In this paper magnetoplasma oscillations of the wave-guided type in semiconductor superlattices are studied. We shall demonstrate that oscillation frequencies much lower than the electron cyclotron frequency and depending inversely on the external magnetic field can exist, under certain conditions, in a periodic array of *homogeneous* 2D electron layers. Let us note that for the case of a zero magnetic field modes of the wave-guided type in a superlattice were previously described by Korzh and Kosevich [14].

2. The superlattice structure model

The model we have adopted to describe the superlattice structure consists of a periodic array of strictly 2D electron layers at the positions $z = nd$, where $n = 0, \pm 1, \pm 2, \ldots$ is the layer index, and d is the distance between adjacent layers. The array is embedded in a homogeneous dielectric medium with dielectric permittivity ε. A static magnetic field H_0 is applied along the direction perpendicular to the planes. The vibration spectrum of the described system can be obtained by writing the general solution of the wave equation in the regions between the electron layers, assuming that the electric field of the collective excitations forms a Bloch wave, and imposing the standard electromagnetic boundary conditions at each of the 2D electron layers. In the local limit $k \ll k_F$ (k_F being the Fermi wavevector of a 2D electron gas, and k being the component of the wavevector of the collective excitation parallel to the layers) the resulting dispersion relation is given by

$$(\omega^2 + (\Omega c\alpha/2\varepsilon)S(\alpha, q))(1 + (\Omega/2c\alpha)S(\alpha, q)) = \omega_H^2 \tag{1}$$

where $\alpha = (\omega^2\varepsilon/c^2 - k^2)^{1/2}$, $\Omega = 4\pi e^2\eta_0/m^*c$, $\omega_H = eH_0/m^*c$, $m^* = m(1 + \mathrm{i}/\omega\tau)$, e, m, η_0 and τ are, respectively, the charge, the effective mass, the surface density and the relaxation time of the 2D carriers. The superlattice structure factor $S(\alpha, q)$ is given by the following expression

$$S(\alpha, q) = \sin(\alpha d)/[\cos(\alpha d) - \cos(qd)]$$

where $\hbar q$ is the quasi-impulse of the collective excitations along the superlattice axis [15]. Let us note that the dispersion relation (1) emerges from the microscopic quantised approach given in [4] when the spatial dispersion in the components of the dynamical magnetoconductivity tensor of the electron system is neglected and it is assumed that between ω and k the relation $\omega^2 > c^2k^2/\varepsilon$ takes place.

We have written the dispersion relation (1) in a form, which permits us to describe the electromagnetic waves of the wave-guided type interacting with the periodic array of 2D electron layers (for these modes $\alpha^2 > 0$). In the following discussion we assume that the frequencies under consideration are much higher than the inverse relaxation time of the 2D carriers ($\omega\tau \gg 1$).

3. Analysis of the model

First of all we analyse (1) in the close vicinity of the line $\omega = ck/\varepsilon^{1/2}$. In this case the coupling between layers is strong (i.e. $\alpha d \ll 1$) and (1) can be written in the form

$$\omega^2 = \omega_0^2 + \{(\alpha^2 d^2/2 - 2\sin^2(qd/2))^{-1} - \tfrac{1}{2}\omega_H^2 d^2\, \varepsilon(s_0^2/c^2 + 2\sin^2(qd/2))^{-2}\}\, \omega_{\mathrm{pl}}^2\, \tfrac{1}{2}\alpha^2 d^2. \tag{2}$$

Here $\omega_{\mathrm{pl}} = (\Omega c/\varepsilon d)^{1/2}$ is the frequency of the 3D plasmon with effective bulk density $n_{\mathrm{B}} = \eta_0/d$; $s_0 = (\Omega c d/\varepsilon)^{1/2}$ is the characteristic phase velocity of an 'acoustic' 2D plasmon [16]; finally, the squared frequency ω_0^2 is given by

$$\omega_0^2 = 2\sin^2(qd/2)\omega_H^2/[2\sin^2(qd/2) + (s_0/c)^2\varepsilon]. \tag{3}$$

For $qd = 0$ (2) takes the following form

$$\omega^2 = \omega_{\mathrm{pl}}^2 - (\omega_H/\omega_{\mathrm{pl}})^2(c\alpha)^2/\varepsilon. \tag{4}$$

If we put here $\alpha = 0$, we can find the frequency at the point where the curve $\omega = \omega(k, q)$ intersects the line $\omega = ck/\varepsilon^{1/2}$. At this point the frequency of the excitation is $\omega = \omega_{\mathrm{pl}}$, i.e. the $qd = 0$ mode in the strong coupling limit is just like a 3D plasmon, propagating with a group velocity given by

$$v_0 = (\partial\omega/\partial k)_{\alpha=o} = (\omega_H^2/(\omega_H^2 + \omega_{\mathrm{pl}}^2))c/\varepsilon^{1/2}. \tag{5}$$

We can see that under the condition $\omega_H \ll \omega_{\mathrm{pl}}$ this group velocity depends on the external magnetic field: in this case $v_0 = \omega_H^2 d/\Omega$ ($v_0 \ll c$). On the other hand, if the parameter relation $\omega_H \gg \omega_{\mathrm{pl}}$ takes place, the group velocity of the $qd = 0$ mode coincides with the velocity of propagation of an electromagnetic wave in a medium with dielectric permittivity ε ($v_0 = c/\varepsilon^{1/2}$). As a result of the above consideration, we can state that in the close vicinity of the point $\alpha = 0$, $\omega = \omega_{\mathrm{pl}}$ a slowly increasing oscillation frequency is observed when the electron cyclotron frequency is lower than ω_{pl}. Let us remark that this oscillation frequency depends on the squared external magnetic field.

For $qd \neq 0$ (2) takes the form

$$\omega^2 = \omega_0^2 - s^2\alpha^2. \tag{6}$$

Here s is a characteristic magnitude with dimensions of velocity. It is given by the following expression

$$s^2 = \{[2\sin^2(qd/2)]^{-1} + (\omega_H^2 d^2/2c^2)\varepsilon[2\sin^2(qd/2) + (s_0/c)^2\varepsilon]^{-2}\}s_0^2. \tag{7}$$

We see that in the case of oblique incidence (i.e. for finite wavevectors in the layer planes and for $qd \neq 0$) the frequency ω_0, defined by (3), represents the frequency at the point where the curve $\omega = \omega(k, q)$ intersects the line $\alpha = 0$. If the condition $\Omega d \gg 2c$ is satisfied, then ω_0 lies below the electron cyclotron frequency ω_H. On the other hand, if

$\Omega d \ll 2c$ and $\Omega \ll cq$, the frequency at the intersection point is close to the electron cyclotron frequency ($\omega_0 \simeq \omega_H$).

It is easy to show that the group velocity of the $qd \neq 0$ mode at the point $\alpha = 0$, $\omega = \omega_0$ is

$$v_0 = (2\sin^2(qd/2)/[2\sin^2(qd/2) + (s/c)^2\varepsilon)](s/c)s/\varepsilon^{1/2}. \tag{8}$$

If $\Omega d \ll 2c$ and $\Omega \ll cq$, then the group velocity v_0 is much less than $c/\varepsilon^{1/2}$. For these parameter relations the character of v_0 depends on the dimensionless quantity $\xi_H = \omega_H\varepsilon^{1/2}d/c$. (i) If $\xi_H \gg 1$, then the group velocity v_0 is proportional to H_0. (ii) If $\xi_H \ll 1$, then v_0 does not depend on the external magnetic field. In both cases (i), (ii) we observe a small linear-in-k and increasing oscillation frequency above ω_H.

4. Further analysis

Let us now consider the frequency splitting and the shift of homogeneous ($k = 0$) wave-guided magnetoplasma oscillations of a layered electron system. In this case (1) can be transformed to the form

$$\sin(\xi)/[\cos(\xi) - \cos(qd)] = -(2c/\Omega d)(\xi \pm \xi_H) \tag{9}$$

where $\xi = \omega\varepsilon^{1/2}d/c$. A similar Kronig–Penney-like expression has been derived previously in the quantum regime [4] and in the semi-classical approach in a superlattice system with finite electron layer thickness [5]. In the particular case $qd = \pi/2$ the relation (9) coincides with the dispersion relation describing the shift and the splitting of homogeneous magnetoplasma oscillations of a *single* 2D electron layer in a symmetric screened system [13]. This coincidence is due to the fact that the electrodynamics of a single 2D electron layer in a symmetric screened system is equivalent to the electrodynamics of a periodic array of 2D electron layers with surface currents oscillating with a difference of phase of $\pi/2$ between adjacent layers.

In the case when $qd \ll 1$, and assuming that the excitation frequency satisfies the inequality $\xi \ll 1$, the dispersion relation (9) takes the form

$$\omega = \omega_H(cq)^2/[\omega_{\mathrm{pl}}^2 + (cq)^2] \tag{10}$$

which is just the dispersion relation for the helicon propagating along the superlattice axis [4]. Under the assumption $\omega_{\mathrm{pl}} \gg cq$, the helicon frequency is $\omega \sim (c^2q^2)/\sigma_H$, where $\sigma_H = ec\eta_0/H_0$ is the Hall conductivity of the 2D electron gas. Under the conditions of the quantum Hall effect (see for example [17]) the helicon frequency in a superlattice shows plateaus. This result has been widely discussed elsewhere [4–10].

Let us analyse (9) under another parameter condition $\omega_H \gg cq/\varepsilon^{1/2}$. In order to understand the distribution of the starting points of the dispersion curves of the wave-guided type let us analyse the graphical solutions of (9) with the aid of the intersection points of the functions $F_1(\xi) = \sin(\xi)/[\cos(\xi) - \cos(qd)]$ and $F_2(\xi) = -(2c/\Omega d)(\xi \pm \xi_H)$ (figure 1). We see that the interaction between wave-guided electromagnetic modes and collective excitations of the layered system leads to the appearance of a shift and splitting of the frequencies corresponding to the asymptotics of the function $F_1(\xi)$ ($1/F_1(\xi_n) = 0$ for $\omega_n = (2\pi n + qd)c/d\varepsilon^{1/2}$, $n = 0, 1, \ldots$). The behaviour of the indicated shift and splitting depends on the positions of the frequencies ω_n with respect to the characteristic frequency ω_H:

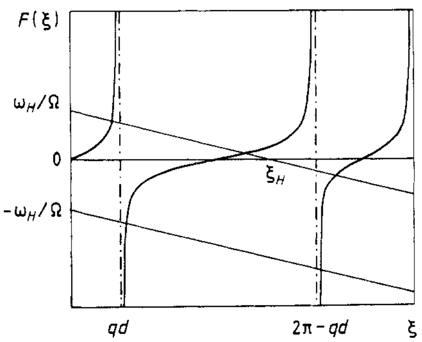

Figure 1. Graphical solution of (9). The thick (thin) continuous curves represent the functions $F_1(\xi)$, ($F_2(\xi)$) defined in the text. The chain lines represent the asymptotics of $F_1(\xi)$.

(i) If $\omega_n < \omega_H$, then one of the split frequencies is localised above the frequency ω_n. On the other hand, the second split frequency lies below ω_n.

(ii) If $\omega_n > \omega_H$, then both split frequencies are localised above ω_n. In this case the gap existing between the split frequencies decreases with the increase of the number n characterising the wave-guided mode.

We see also, that as a result of the interaction of the electromagnetic waves with the cyclotron oscillations of the layered electron gas, the electron cyclotron resonance frequency ω_H shows a shift, but it does not undergo splitting. We see that the shift of ω_H is positive (negative) if ω_H lies below (above) the corresponding nearest zero of the function $F_1(\xi)$. In the case when ω_H coincides with this zero, there is no shift of the cyclotron frequency.

Let us assume that the parameter relations $\omega_H \gg cq/\varepsilon^{1/2}$ and $\Omega \ll \omega_H$ take place. Then (9) exhibits solutions corresponding to frequencies much lower than the electron cyclotron frequency, but showing a different, in comparison with helicon modes (10), dependence on the external magnetic field. These frequencies can be expressed with the aid of a series expansion on the small dimensionless parameter Ω/ω_H. In the first approximation we have

$$\omega = (2\pi n - qd)c/d\varepsilon^{1/2} \pm (2\pi\sigma_H/\varepsilon d) \qquad n = 1, 2, 3, \ldots \tag{11}$$

$$\omega = (2\pi n + qd)c/d\varepsilon^{1/2} \pm (2\pi\sigma_H/\varepsilon d) \qquad n = 0, 1, 2, \ldots. \tag{11a}$$

We see that for $\Omega \ll \omega_H$ a strong magnetic field (in the sense that $\omega_H \gg cq/\varepsilon^{1/2}$) causes a frequency shift and splitting of the starting points of the wave-guided modes in a periodic layered system, depending *inversely* on the external magnetic field H_0 and the period of the array of 2D electron layers d. Since these shift and splitting values are directly proportional to the Hall conductivity, they exhibit plateaus concomitant with the plateaus under the conditions of the quantum Hall effect.

It is necessary to remark that in the case of resonance interaction of homogeneous ($k = 0$) electromagnetic wave-guided modes and cyclotron oscillations of the electron system, the frequency splitting is proportional to the square root of the small parameter Ω/ω_H: $\Delta\omega/\omega \sim (\Omega/\omega_H)^{1/2}$. In order to demonstrate this fact let us assume that $\omega_H = (2\pi - qd)c/d\varepsilon^{1/2}$. In this case the Kronig–Penney-like dispersion relation (9) can be

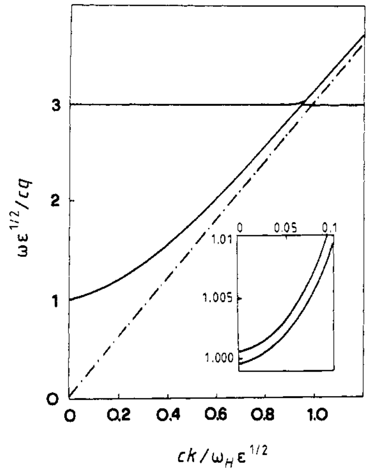

Figure 2. Dispersion curves of the lowest branches of the magnetoplasma oscillations in a periodic array of 2D electron layers: $\Omega d/c = 3 \times 10^{-3}$, $\Omega/\omega_H = 1 \times 10^{-3}$, $qd = \pi/3$, $\eta_0 = 1 \times 10^{11}\,\mathrm{cm}^{-2}$, $m = 1 \times 10^{-28}\,\mathrm{g}$, $\varepsilon = 13$, $d = 5 \times 10^{-4}\,\mathrm{cm}$. The chain line represents the dispersion law $\omega = ck/\varepsilon^{1/2}$. Inset: splitting of lowest wave-guided mode.

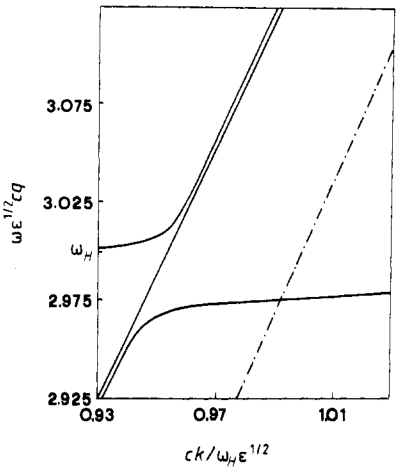

Figure 3. Same as figure 2: details of the oscillation branches in the vicinity of the intercept of straight lines $\alpha = 0$ and $\omega = \omega_0$.

written in the form $(\omega - \omega_H)(\omega \pm \omega_H) = \Omega\omega_H/\pi\varepsilon^{1/2}$. The minus sign corresponds to the frequency

$$\omega = \omega_H(1 \pm (\pi^{1/2}\varepsilon^{1/4})^{-1}(\Omega/\omega_H)^{1/2})$$

which confirms the above mentioned statement that the frequency splitting is proportional to the square root of the small parameter. A similar effect of an increase in resonance splitting was first predicted by V M Agranovich for the case of resonance between excitons in thin dielectric films and surface polaritons (for a review see [18]). For low-dimensional electron systems in an external magnetic field resonance splitting increasing has been discussed in [13, 19, 20].

5. Observations

The deformation (due to the action of a strong magnetic field) of the lowest wave-guided modes interacting with the layered electron system is shown in figure 2. The splitting of the lowest mode in the long-wavelength limit is shown in the inset. For values of the in-plane wavevector corresponding to the limit $kd \ll 1$ the dependence of the split frequencies on the wavevector $\boldsymbol{k}$ can be described with the aid of the following expression:

$$\omega_\pm = (cq/\varepsilon^{1/2})(1 + 2(kd/\pi)^2) \pm \Omega c/2\omega_H\varepsilon d. \tag{12}$$

We see that the split frequencies increase with the square of the component of the wavevector $\boldsymbol{k}$ perpendicular to the external magnetic field.

The vicinity of intercept of straight lines $\omega = \omega_0$ and $\omega = ck/\varepsilon^{1/2}$ is shown in figure 3. We see that with the increasing of the wavevector $\boldsymbol{k}$ the dispersion curve describing the lowest split branch shows a kink when it approximates the frequency ω_0. At $\omega = \omega_0$ this curve intersects the line $\omega = ck/\varepsilon^{1/2}$, enters the region $\alpha^2 > 0$ and continuously transforms into the dispersion law for the plasma oscillations of a layered electron gas. On the other hand, the highest of the split branches does not show kink and does not intersect the line $\omega = ck/\varepsilon^{1/2}$, but approximates it with the increasing of $\boldsymbol{k}$. We see also, that the curve describing coupled cyclotron wave-guided modes shows a small increase in $\boldsymbol{k}$ when $k < \omega_H \varepsilon^{1/2}/c$, but it shows a kink when $\boldsymbol{k}$ reaches the vicinity of the line $\omega = ck/\varepsilon^{1/2}$. Therefore, we can state that in a system with small concentration of the 2D carriers (in the sense that $\Omega d/c \ll 1$) the behaviour of the dispersion curves can be explained as a result of the intersection of the wave-guided dispersion curves with the dispersion curve corresponding to the cyclotron oscillations of a layered electron gas.

6. Conclusions

The dispersion relation for magnetoplasma oscillations of the wave-guided type in semiconductor superlattices has been presented. The behaviour of these modes in the close vicinity of the line $\omega = ck/\varepsilon^{1/2}$ has been discussed. The frequency shift and splitting caused by a strong magnetic field has been studied.

We have demonstrated that resonance modes whose frequencies are smaller than the cyclotron frequency can propagate, under relevant conditions, even in the case of oblique incidence (i.e. by including finite wavevectors in the layer planes).

In discussing the perpendicular propagation of the collective modes we observed a low frequency mode depending inversely on the external magnetic field. At this point it is necessary to outline that, although both helicon and wave-guided-type modes are contained in the same Kronig–Penney-like expression, they have a different physical origin because these resonances take place under different parameter relations. This situation leads to the fact that conditions for the observation of plateaus in frequencies (11, 11*a*) are different from those necessary for the observation of the corresponding plateaus in the helicon frequency. In fact, as Vagner and Bergman have shown [10], in a superlattice with a small concentration of electrons, low frequency helicon waves can be observed by employing small values of q under the condition that plasma frequency must be higher than the cyclotron frequency. For the frequency shift and splitting described by (11, 11*a*) the last limitation is lifted because the condition $\omega_H \gg \Omega$ is essential for the definition of such shift and splitting. In order to satisfy the parameter relation $\omega_H \gg cq/\varepsilon^{1/2}$ we can take sufficiently high magnetic fields: in this case it is not necessary to employ small values of q. This means that it is not essential to use a superlattice with a large number of layers in order to observe the described modes (11, 11*a*).

We should mention an important difference between 'edge' modes and electromagnetic excitations described by (9). For the description of the latter it is necessary to take the retarded effects of the electromagnetic waves into account. On the other hand, the existence of 'edge' magnetoplasmons is connected essentially with the loss of translation invariance of the electron system. It is clear that the behaviour of 'edge' modes does not essentially depend on the retarded effects.

The frequency splitting described by (11, 11*a*) is associated with the Faraday effect for electromagnetic waves propagating along the direction of the magnetic field perpendicular to the electron layer planes. Let us remark that the damping of the magnetoplasma oscillations in a layered system for $\omega \ll \omega_H$, $\omega_H \tau \gg 1$ is determined principally by the dissipative conductivity of a 2D electron gas: $\sigma_{xx} \sim \sigma_{xy}/(\omega_H \tau) \ll \sigma_{xy}$, i.e. the splitting of the resonance lines exceeds their width.

Acknowledgments

One of the authors (JCG) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

References

[1] Cottam M G and Tilley D R 1989 *Introduction to Surface and Superlattice Excitations* (Cambridge: Cambridge University Press) p 333
[2] Cardona M and Guntherodt G (eds) 1989 *Light Scattering in Solids V: Superlattices and other Structures* (Berlin: Springer) p 351
[3] Mizuno J, Kobayashi M and Yokota I 1975 *J. Phys. Soc. Japan* **39** 18
[4] Tselis A, Gonzales de la Cruz G and Quinn J J 1983 *Solid State Commun.* **47** 43
Tselis A and Quinn J J 1984 *Phys. Rev.* B **29** 2021; 1984 *Phys. Rev.* B **29** 3318
[5] Babiker M 1987 *Solid State Commun.* **64** 983
[6] Maan J C, Altarelli M, Sigg H, Wyder P, Chang L L and Esaki L 1982 *Surf. Sci.* **113** 347
[7] Wendler L and Kaganov M I 1986 *Zh. Eksp. Teor. Fiz. Pis. Red.* **44** 345; 1986 *Phys. Status Solidi* b **138** K33; 1987 *Phys. Status Solidi* b **142** K63
[8] Kushwaha M S 1986 *Phys. Status Solidi* b **136** 757
[9] Achar B N 1987 *Phys. Status Solidi* b **140** K37
Achar B N and Liu C 1989 *Phys. Rev.* B **40** 8002
[10] Vagner I D and Bergman D 1987 *Phys. Rev.* B **35** 9856
[11] Glattli D C, Andrei E Y, Deville G, Poitrenaud J and Williams F I B 1985 *Phys. Rev. Lett.* **54** 1710
[12] Mast P B, Dahm A J and Fetter A L 1985 *Phys. Rev. Lett.* **54** 1706
Wu J W, Hawrylak P and Quinn J J 1985 *Phys. Rev. Lett.* **55** 1706
Volkov V A and Mikhailov S A 1988 *Sov. Phys.-JETP* **67** 1639
[13] Kosevich Yu A, Kosevich A M and Granada J C 1988 *Phys. Lett.* **127A** 52; 1988 *Fiz. Nizk. Temp.* **14** 926
[14] Korzh S A and Kosevich A M 1982 *Fiz. Nizk. Temp.* **7** 1382
[15] Fetter A L 1974 *Ann. Phys., NY* **88** 1
[16] Monarkha Yu P 1977 *Fiz. Nizk. Temp.* **3** 1459
[17] Prange R E and Girvin S M (eds) 1987 *The Quantum Hall Effect* (New York: Springer) p 419
[18] Agranovich V M and Ginzburg V L 1984 *Crystal Optics with Spatial Dispersion, and Excitons* (Berlin: Springer) p 441
[19] Kushwaha M and Halevi P 1987 *Solid State Commun.* **64** 1405; 1988 *Phys. Rev.* B **38** 12428
[20] Granada J C 1989 *ICTP Preprint* IC/89/370